Zadanie 6. Nieporządek...

permutacja $\pi : \{1,2,\ldots,n\} \to \underbrace{\{1,2,\ldots,n\}}_{A_n}$ taka że $\forall \pi(i) \neq i$, $i \in \{1,2,\ldots,n\}$

$d_{n+1} = n(d_n + d_{n-1})$ $\qquad d_0 = 1,\ d_1 = 0,\ d_2 = 1$

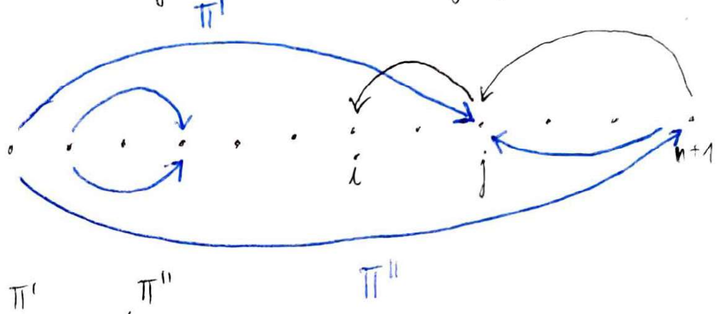
Rozważmy możliwe wartości $\pi(n+1)$ — $\{1,2,\ldots,n\}$. Niech $\pi(n+1) = j$.

1° $\pi(j) = n+1$ Wtedy $A_n^{-j} = A_n \setminus \{j\}$. Zauważmy, że $\pi : A_n^{-j} \to A_n^{-j}$, oraz π jest nieporządkiem na A_n^{-j}, czyli dla ustalonego j takich permutacji jest d_{n-1}.

2° $\pi(j) \neq n+1$ Niech $\pi(j) = i$. Pokażemy bijekcję między π' oraz π'':

π' – nieporządek na A_n

π'' – nieporządek na A_{n+1} taki, że $\pi''(n+j)$, ale $\pi''(j) \neq n+1$



$$\overset{\pi'}{\swarrow}\quad \overset{\pi''}{\swarrow}$$

$$f : P_n \to P_{n+1}^{j}, \quad \sigma = f(\pi)$$

$$\sigma(i) = \begin{cases} \pi(i) & : i \leq n \text{ oraz } \pi(i) \neq j \\ n+1 & : \pi(i) = j \\ j & : i = n+1 \end{cases}$$

$$d_n = \sum_{i=0}^{n} \frac{(-1)^i}{i!} \approx \frac{n!}{e}$$

$$d_{n+1} = n \cdot d_n + n \cdot d_{n-1} \quad / \cdot \frac{x^n}{n!} \text{ (dla } n \geq 0)$$

$$\frac{d_{n+1} x^n}{n!} = \frac{n \cdot d_n \cdot x^n}{n!} + \frac{n \cdot d_{n-1} x^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{d_{n+1} x^n}{n!} = \sum_{n=1}^{\infty} \left(\frac{n d_n x^n}{n!} + \frac{n d_{n-1} x^n}{n!} \right)$$

$$D_e'(x) = \left(D_e(x) - d_0\right)' = x \cdot \sum_{n=1}^{\infty} \frac{n d_n x^{n-1}}{n!} + x \cdot \sum_{n=1}^{\infty} \frac{d_{n-1} x^{n-1}}{(n-1)!}$$

$$D_e'(x) = x \cdot D_e'(x) + x D_e(x)$$

$$(1-x) D_e'(x) = x \cdot D_e(x)$$

$$\frac{D_e'(x)}{D_e(x)} = \frac{x}{1-x}$$

$$\ln D_e(x) = \int \frac{x}{1-x} dx + C$$

$$D_e(x) = e^{\int \frac{x}{1-x} dx + C} = \left| \begin{array}{l} y = 1-x \\ dy = -dx \end{array} \right| =$$

$$= e^{1-x-\ln(1-x)+C} = \frac{e^{1-x} \cdot e^{C}}{1-x}$$

$$\left. \begin{array}{l} D_e(0) = \frac{e^1 \cdot e^C}{1} = e^{1+C} \\ D_e(0) = d_0 = 1 \end{array} \right\} \quad \begin{array}{l} C = -1 \\ D_e(x) = \frac{e^{-x}}{1-x} \end{array}$$

$$\frac{f'(x)}{f(x)} = g(x) \qquad \left\{ \begin{array}{l} y = f(x) \\ dy = f'(x) dx \end{array} \right.$$

$$\int \frac{f'(x)}{f(x)} dx = \int g(x) dx + C$$

$$\|$$

$$\int \frac{1}{y} dy = \ln y = \ln f(x)$$

Zadanie 2. $A(x) = \sum_{n=0}^{\infty} a_n x^n$

$$\sum_{n=0}^{\infty} a_{2n} x^n = \frac{A(\sqrt{x}) + A(-\sqrt{x})}{2}$$

$$\sum_{n=0}^{\infty} a_{3n} x^n = \frac{A(\sqrt[3]{x}) + A\left(\sqrt[3]{x} \cdot \frac{-1-i\sqrt{3}}{2}\right) + A\left(\sqrt[3]{x} \cdot \frac{-1+i\sqrt{3}}{2}\right)}{3}$$

$$z^3 = 1$$

$$(z-1)(z^2+z+1) = 0$$

$$z = \frac{-1 \pm i\sqrt{3}}{2} \vee z = 1$$

Wzór na funkcję tworzącą d_{kn} (niepełny):

$$x_{k,n} = e^{i \cdot 2\pi \cdot \frac{k}{n}}, \quad S_j = \sum_{k=0}^{n-1} x_{k,n}^j$$

$1°$ $n \mid j$: $S_j = n-1$

$2°$ $n \nmid j$: $S_j = \dfrac{e^{i \cdot 2\pi \cdot \frac{j}{n}} - 1}{e^{i \cdot 2\pi \cdot \frac{k}{n}} - 1}$?

Zadanie 11. c

$$\prod_{k=1}^{\infty}\left(1+x^{2k-1}\right)$$

Zadanie 11. d

$$\prod_{k=0}^{\infty}\left(1+x^{2^k}\right) = \sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$